Green's functions

How to obtain all of them from those simulated at $v = c$?

Consider the focused transport equation (FTE) with adiabatic energy changes neglected:

$$\frac{\partial f}{\partial t} + v\mu\frac{\partial f}{\partial z} + \frac{1-\mu^2}{2L}v\frac{\partial f}{\partial \mu} - \frac{\partial}{\partial \mu}\frac{1-\mu^2}{2\lambda}v\frac{\partial f}{\partial \mu} = \frac{\delta(p-p_0)H(\mu)}{2\pi p^2}\cdot\frac{\delta(z-z_0)}{A}\cdot\delta(t)$$

Here, the right-hand side is normalized to 1 particle injected into the flux tube at time $t = 0$, at location $z = z_0$, at momentum p_0 and at pitch-cosine $0 < \mu < 1$.

$f = dN/d^3x\, d^3p$ = particle distribution function in 6-D phase space
A = flux tube cross-sectional area
$L = A/(\partial A/\partial z)$ = focusing length
λ = scattering mean free path

Note that we have assumed isotropic scattering (although the conclusions are not affected by this assumption).

Instead of f, we can consider the differential intensity, $dI/dE = p^2 f$, and multiply the FTE by p^2/v to get

$$\frac{\partial}{\partial s}\frac{dI}{dE} + \mu\frac{\partial}{\partial z}\frac{dI}{dE} + \frac{1-\mu^2}{2L}\frac{\partial}{\partial \mu}\frac{dI}{dE} - \frac{\partial}{\partial \mu}\left(\frac{1-\mu^2}{2\lambda}\frac{\partial}{\partial \mu}\frac{dI}{dE}\right) = \frac{\delta(p-p_0)H(\mu)}{2\pi}\cdot\frac{\delta(z-z_0)}{A}\cdot\delta(s)$$

where $s = vt$ is the path length traversed by the particle.

For a mono-energetic simulation, we give the registered number of particles at the observer's location integrated over energy. This, the recorded quantity is

$$I = \int dE\, dI/dE$$

FTE for *I* reads

$$\frac{\partial I}{\partial s} + \mu \frac{\partial I}{\partial z} + \frac{1-\mu^2}{2L}\frac{\partial I}{\partial \mu} - \frac{\partial}{\partial \mu}\frac{1-\mu^2}{2\lambda}\frac{\partial I}{\partial \mu} = \frac{v_0 H(\mu)}{2\pi} \cdot \frac{\delta(z-z_0)}{A} \cdot \delta(s)$$

where

$$\int dE\, \delta(p-p_0) = \int v\, \delta(p-p_0)\, dp = v_0$$

has been applied.

Using the scaled quantity $j = I\, c/v_0$ we can get rid of speed in the equation completely:

$$\frac{\partial j}{\partial s} + \mu \frac{\partial j}{\partial z} + \frac{1-\mu^2}{2L}\frac{\partial j}{\partial \mu} - \frac{\partial}{\partial \mu}\frac{1-\mu^2}{2\lambda}\frac{\partial j}{\partial \mu} = \frac{c H(\mu)}{2\pi} \cdot \frac{\delta(z-z_0)}{A} \cdot \delta(s)$$

This is valid for all values of v_0 regardless of the speed we use to obtain a solution. Thus

$$j(s,z,\mu) = I(s,z,\mu,c)$$

We got

$$\frac{\partial I}{\partial s}+\mu\frac{\partial I}{\partial z}+\frac{1-\mu^2}{2L}\frac{\partial I}{\partial \mu}-\frac{\partial}{\partial \mu}\frac{1-\mu^2}{2\lambda}\frac{\partial I}{\partial \mu}=\frac{v_0 H(\mu)}{2\pi}\cdot\frac{\delta(z-z_0)}{A}\cdot\delta(s)$$

for $I=\int dE\, dI/dE$ and

$$\frac{\partial j}{\partial s}+\mu\frac{\partial j}{\partial z}+\frac{1-\mu^2}{2L}\frac{\partial j}{\partial \mu}-\frac{\partial}{\partial \mu}\frac{1-\mu^2}{2\lambda}\frac{\partial j}{\partial \mu}=\frac{cH(\mu)}{2\pi}\cdot\frac{\delta(z-z_0)}{A}\cdot\delta(s)$$

for $j=Ic/v_0=I(s,z,\mu,c)$

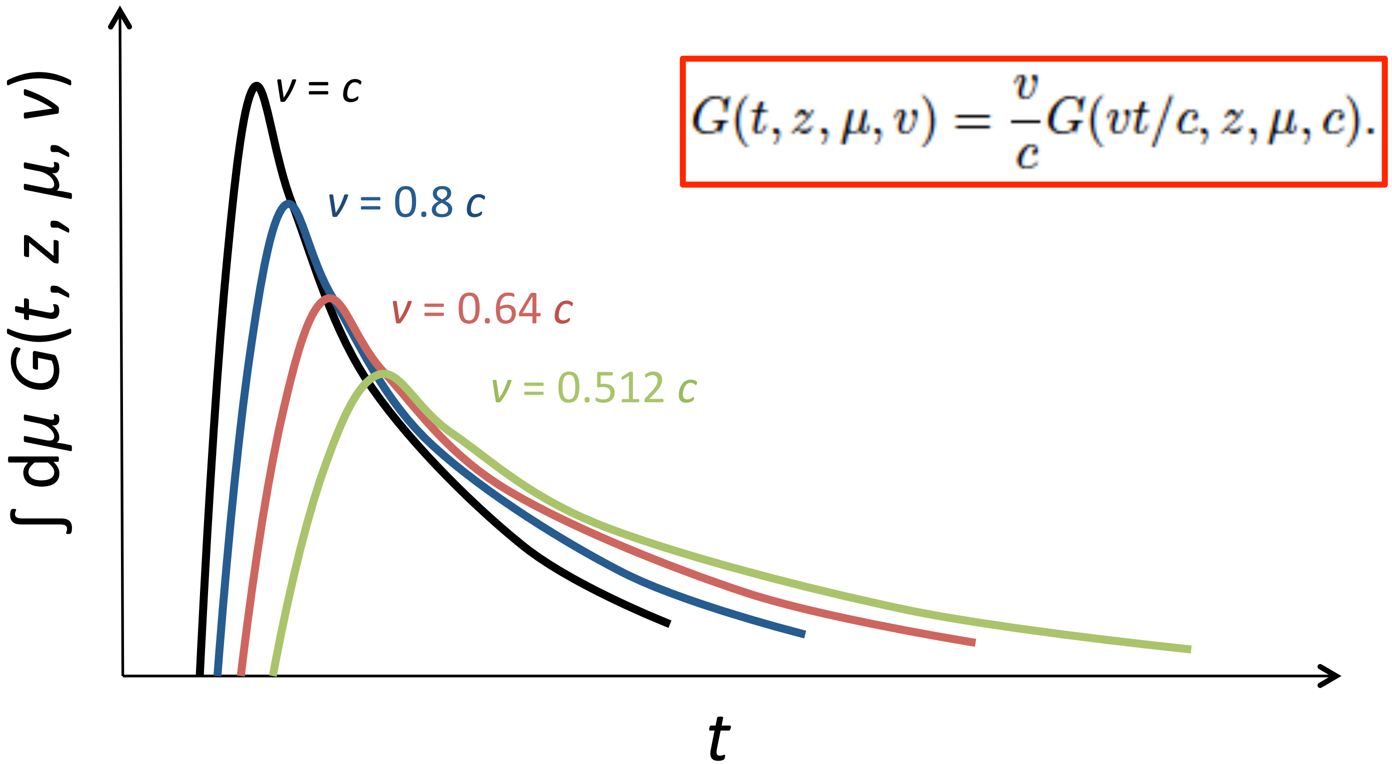
Thus, the Green's function, with *t* replacing *s* as the independent variable, is

$$G(t,z,\mu,v)=I(s,z,\mu,v)=\frac{v}{c}j(s,z,\mu)=\frac{v}{c}I(s,z,\mu,c)=\frac{v}{c}G(s/c,z,\mu,c)$$

and since *s* = *vt*, we get

$$\boxed{G(t,z,\mu,v)=\frac{v}{c}G(vt/c,z,\mu,c).}$$

Therefore, simulating the Green's function at *v* = *c* allows one to obtain the solution at an arbitrary speed, *v*, provided that adiabatic deceleration can be neglected.



v = c
v = 0.8 c
v = 0.64 c
v = 0.512 c
∫ dμ G(t, z, μ, v)
t
$G(t,z,\mu,v) = \frac{v}{c} G(vt/c, z, \mu, c).$