Particle Propagation

Neus Agueda
University of Barcelona

30 August 2016

Understanding Solar Eruptions and Extreme Space Weather Events.
The physics behind

HESPERIA
SUMMER SCHOOL

Outline

- What are SEPs?
    - and why do we care?
- Charged particle propagation
    - homogeneous fields
    - inhomogeneous fields: drifts, first adiabatic invariant
- How do SEPs propagate in interpanetary space?
    - the interplanetary magnetic field
    - particle transport models
    - the power of convolution
- SEP release timescales and transport parameters from
    - forward/inverse modeling

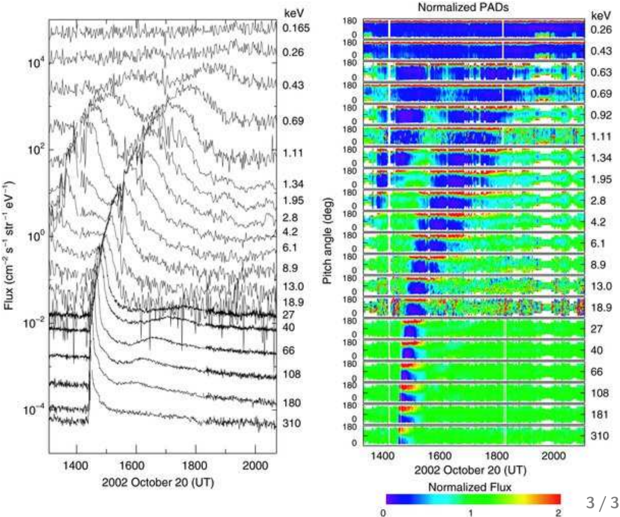
What are SEPs?

Solar Energetic Particle Events

Seen as increases in counting rates of ions and/or electrons of energies usually above several keV and up to GeV in the most energetic cases

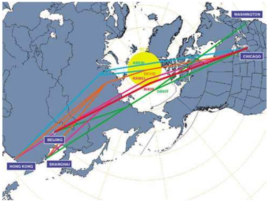
- *solar*: assumed to originate from solar eruptions
- *energetic*: from a few hundred keV to some GeV
- *particles*: protons, electrons and heavy ions

(Wang et al. 2011)

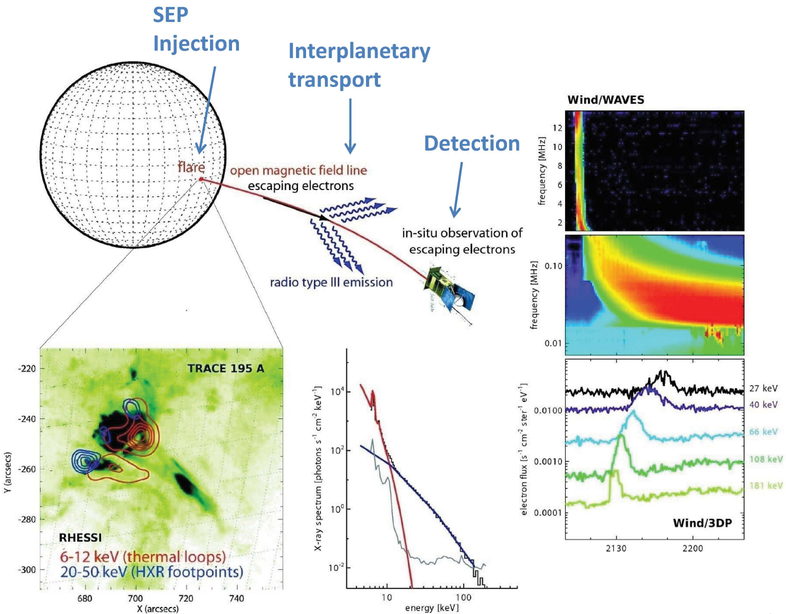


Why do we care about SEP events?

- Earth effects
    - radiation hazard for Earth-orbiting spacecraft: *degradation, on board electronics malfunction, mission loss*
    - threat for astronauts on exploratory missions
    - part of geomagnetic storms which can cause
        - aurora
        - black outs
        - major disturbances of radio communications in polar regions



- A sample of the Sun
    - one of the most accurately measured solar samples
    - if we can just figure out the details of creating them and getting them here



SEP Injection

Interplanetary transport

Detection

Figure credit: S. Krucker

Charged particle propagation

Eqs. of motion:

$$\frac{d\vec{p}}{dt} = q(\vec{E} + \vec{v} \times \vec{B})$$

$$\frac{d\vec{r}}{dt} = \vec{v} = \frac{\vec{p}}{(\gamma m)}$$

where $\gamma = (1 + p^2/m^2c^2)^{1/2}$ is the Lorentz factor of the particle.

Examples:

① $\vec{B} = 0,\ \vec{E} = E_0\hat{z} \longrightarrow \vec{p} = (p_{0x}, p_{0y}, p_{0z} + qE_0 t) \longrightarrow$ acceleration

② $\vec{E} = 0,\ \vec{B} = B_0\hat{z} \longrightarrow \vec{p} = (p_\perp \cos\omega_c t, -p_\perp \sin\omega_c t, p_{||})$

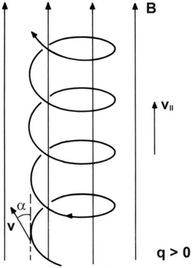
where $\boxed{\omega_c = \frac{qB}{\gamma m}}$ is the **gyro frequency**

and $p = \sqrt{p_{||}^2 + p_\perp^2}$ is **conserved**

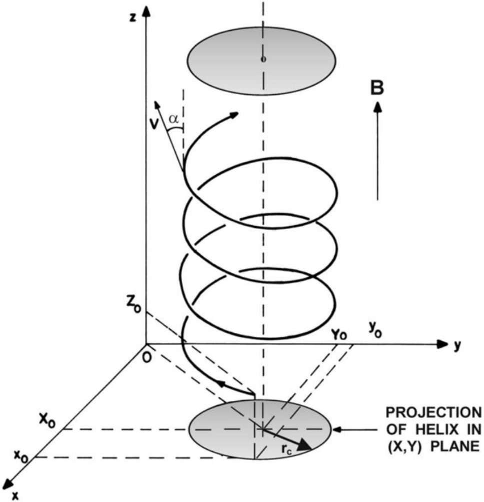
Thus,

$\vec{r} = (x_0 + r_L \sin\omega_c t, y_0 + r_L \cos\omega_c t, z_0 + v_{||} t)$

where $\boxed{r_L = \frac{v_\perp}{\omega_c} = \frac{p_\perp}{qB_0}}$ is the **Larmor radius**



Particle Pitch Angle



- The resulting trajectory of the particle is given by the superposition of
  - a uniform motion along $\vec{B}$ (with constant velocity $\vec{v}_{||}$) and
  - a circular motion in the plane normal to $\vec{B}$ (with constant velocity $\vec{v}_\perp$)
- Hence, the particle describes a **helix**.
- The angle between the direction of motion of the particle and $\vec{B}$ is called **pitch angle**

$$\boxed{\alpha = \sin^{-1}\left(\frac{v_\perp}{v}\right) = \tan^{-1}\left(\frac{v_\perp}{v_{||}}\right)}$$

where v is the total speed of the particle $v^2 = v_{||}^2 + v_\perp^2$

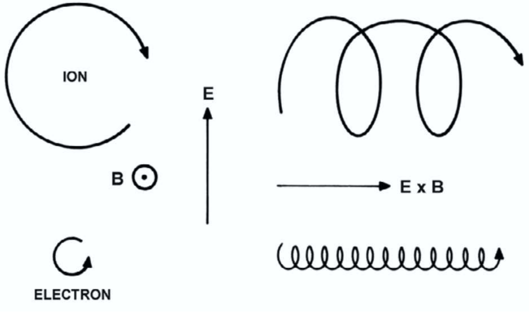
Plasma drift velocity - $\vec{E} \times \vec{B}$ drift

- Consider next a constant electric ($\vec{E}$) and a magnetic ($\vec{B}$) field perpendicular to each other.
- In the direction parallel to $\vec{B}$, we obtain a motion with constant acceleration $q\vec{E_{||}}/m$.
- In the direction perpendicular to $\vec{B}$, in the reference system moving with the constant velocity $\vec{v_E}$, we obtain pure cyclotron motion

$$\boxed{\vec{v_E} = \frac{\vec{E}\times\vec{B}}{B^2}}$$

For any $\vec{F}$:

$$\boxed{\vec{v_F} = \frac{\vec{F}\times\vec{B}}{qB^2}}$$



Nonuniform Magnetic Fields

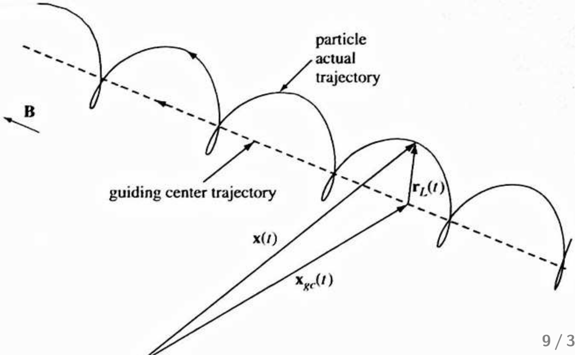
- In a homogeneous B-field, the particle trajectory is a helix
  - The so-called guiding center of the particle follows a straight line parallel to the magnetic field.

- In a weakly inhomogeneous field ($r_L \ll L$) $\longrightarrow$ enough to follow the motion of GC. Governed by
  - gradual drift of the GC across $\vec{B}$
  - change of velocity along $\vec{B}$ $\rightarrow$adiabatic invariants

$$\vec{B}(\vec{r}) = \vec{B_0} + \vec{r} \cdot (\vec{\nabla}\vec{B}) + \ldots$$

$$\delta B = |\vec{r} \cdot (\vec{\nabla}\vec{B})| \ll |\vec{B_0}|$$

Equation of motion:
$m\frac{d\vec{v}}{dt} = q(\vec{v} \times \vec{B_0}) + q\vec{v}^{(0)} \times [\vec{r}^{(0)} \cdot (\vec{\nabla}\vec{B})]$



Gradient drift

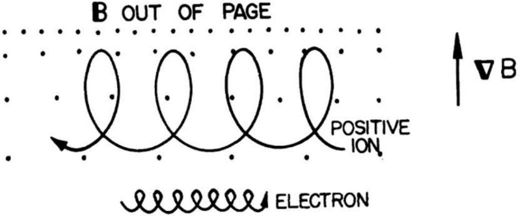
- In a slightly inhomogeneous magnetic field, the Lorentz force averaged over one gyration is

$$\vec{F}_B = -|\vec{m}|\vec{\nabla}B = -\left(\frac{v_\perp p_\perp}{2B}\right)\vec{\nabla}B$$

  where $\vec{m} = -(v_\perp p_\perp/2B)\vec{b}$ is the magnetic moment of the particle.

- This causes the so-called **gradient drift** at velocity

$$\vec{v}_G = \frac{\vec{F}_\perp \times \vec{B}}{qB^2} = -\frac{|\vec{m}|}{q}\frac{(\vec{\nabla}B)\times\vec{B}}{B^2}$$



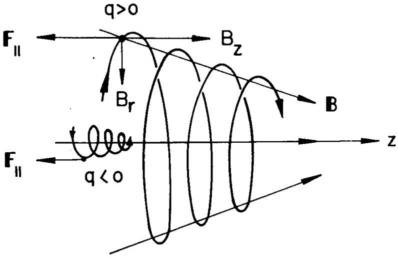
First Adiabatic Invariant

When the spatial variation of $\vec{B}$ inside the particle orbit is small compared with the magnitude of $\vec{B}$, the first adiabatic invariant is conserved

$$\boxed{\frac{\frac{1}{2}mv_\perp^2}{B} = \frac{\sin^2\alpha}{B} = \frac{1-\mu^2}{B} = \text{const.}}$$

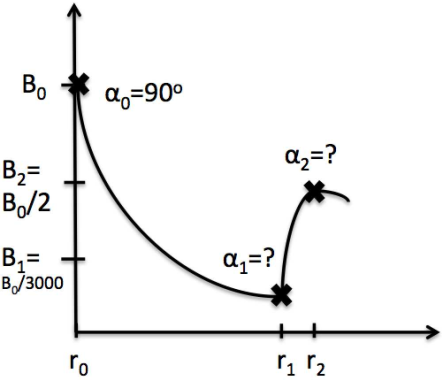
where $\mu = \cos\alpha$ is the particle pitch angle cosine.
This is related to the conservation of magnetic flux $\Phi_m = \int \vec{B} \cdot d\vec{S}$

- **Mirroring:** As the particle moves in a region of converging $\vec{B}$ field ($B\uparrow$), the particle will orbit with increasingly smaller radius ($\alpha\uparrow$).
- **Focusing:** In a diverging $\vec{B}$ field ($B\downarrow$), the particle will orbit with increasingly larger radius ($\alpha\downarrow$).



Exercise

$$\boxed{\frac{\sin^2\alpha}{B}=\text{const.}}$$



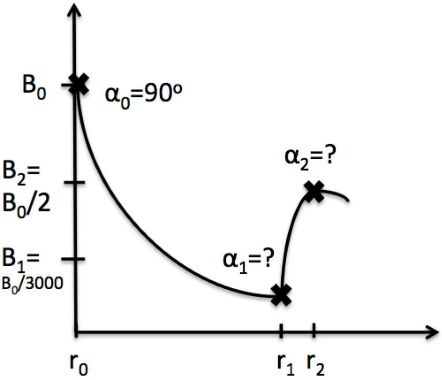
- What is the value of α_1?

- If instead $\alpha_1 = 30^\circ$, would the particle mirror inside $[r_0, r_1]$?

- What is the value of α_2? Will the particle overcome the magnetic barrier?

Exercise

$$\boxed{\frac{\sin^2\alpha}{B} = \text{const.}}$$



$$\sin^2\alpha_1 = \frac{B_1}{B_0}\sin^2\alpha_0 = 1/3000$$
$$\rightarrow \alpha_1 \sim 1^\circ$$

$$\frac{B_1}{B_3} = \sin^2 30^\circ = \frac{1}{4}$$

$$\sin^2\alpha_2 = \frac{B_2}{B_0}\sin^2\alpha_0 = 1/2$$
$$\rightarrow \alpha_1 \sim 45^\circ$$

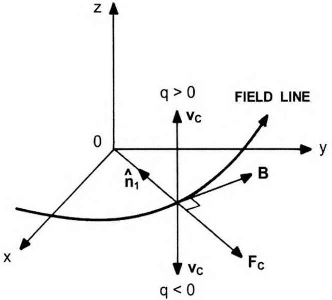
Curvature drift

In weakly inhomogeneous magnetic field, the GC follows the magnetic field lines to the lowest approximation. If the field lines are curved, GC feels the centrifugal force

$$\vec{F}_C = -\frac{mv_{||}^2}{R}\hat{n}_1$$

where R denotes the local radius of curvature of the magnetic field line. The drift associated with this force is

$$\vec{v}_C = \frac{\vec{F}_C \times \vec{B}}{qB^2} = -(\hat{n}_1 \times \vec{B})\frac{mv_{||}^2}{RqB^2}$$



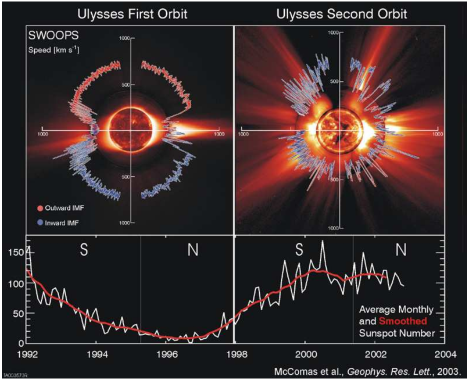
The gradient and the curvature drifts will always appear together ($\vec{\nabla} \times \vec{B} = 0$) pointing in the same direction.

$$\vec{v}_{GC} = -\frac{mv_{\perp}^2}{2qB^3}(\vec{\nabla}B) \times \vec{B} - \frac{mv_{||}^2}{qB^2}[(\vec{B} \cdot \vec{\nabla})\vec{B}] \times \vec{B}$$

The solar wind

- As a consequence of the high temperatures found in the solar corona, the solar atmosphere is not stable but blown away as **solar wind**.

  This thermally-driven flow of ionized, charged plasma constitutes an ever present wind that carves out a cavity, the **heliosphere**, in the surrounding interstellar medium.



McComas et al., *Geophys. Res. Lett.*, 2003.

- The variability of the solar wind in space and time reflects the underlying coronal structures.
- The solar wind carries the solar magnetic field out into the heliosphere.

Two basic types of solar wind

Two distinct types of flow are observed (fast and slow)

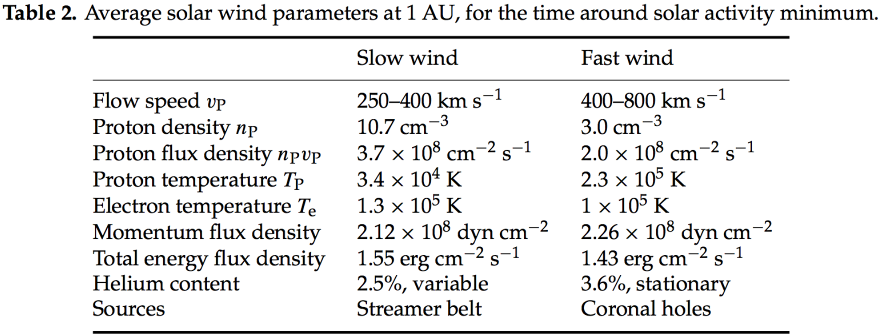


Table 2. Average solar wind parameters at 1 AU, for the time around solar activity minimum.

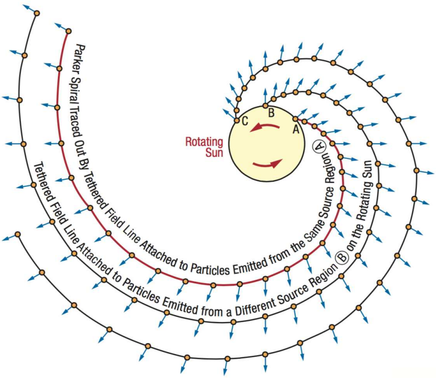
	Slow wind	Fast wind
Flow speed v_P	250–400 km s^{-1}	400–800 km s^{-1}
Proton density n_P	10.7 cm^{-3}	3.0 cm^{-3}
Proton flux density $n_P v_P$	3.7×10^8 cm^{-2} s^{-1}	2.0×10^8 cm^{-2} s^{-1}
Proton temperature T_P	3.4×10^4 K	2.3×10^5 K
Electron temperature T_e	1.3×10^5 K	1×10^5 K
Momentum flux density	2.12×10^8 dyn cm^{-2}	2.26×10^8 dyn cm^{-2}
Total energy flux density	1.55 erg cm^{-2} s^{-1}	1.43 erg cm^{-2} s^{-1}
Helium content	2.5%, variable	3.6%, stationary
Sources	Streamer belt	Coronal holes

Despite their differences, fast and slow solar wind streams have similar total energy flux and momentum flux density.

For $u = 400$ km/s $\longrightarrow \Delta t_{\odot-1\ \mathrm{AU}} \sim 4$ days

The interplanetary magnetic field

- In a highly conducting fluid the magnetic field lines move along exactly with the fluid ($\vec{B}$ is “frozen” in the outflowing solar wind plasma).
- Plasma streams move outward from the Sun along radial lines. But because they are tethered to the rotating Sun, the magnetic field lines that trail behind them are spirals.



- The SEP trajectories are shaped by the **interplanetary magnetic field** (IMF): smooth average Archimedean spiral (Parker 1958) with superimposed irregularities

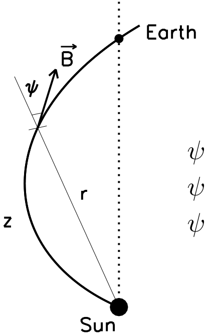
The Parker spiral

In the equatorial plane, the spiral is given by

$$\phi(t) = \phi_0 + \Omega t$$

$$r(t) = r_0 + ut \rightarrow \boxed{r = r_0 + u\frac{\phi-\phi_0}{\Omega}}$$

The **path length**, z ($dz = \sec\psi dr$), along the spiral is $z \sim r$ close to the Sun and $z \sim r^2/2a$ beyond 1 AU.



$$\boxed{\tan\psi = \frac{r\Omega}{u}}$$

$\psi \sim 30°$ at 0.5 AU

$\psi \sim 45°$ at 1.0 AU

$\psi \sim 87°$ at 5.0 AU

The **field strength** is given by

$$\boxed{B(r) = B_0\left(\frac{r_0}{r}\right)^2\sqrt{1+\left(\frac{r}{a}\right)^2}}$$

where $a = u/\Omega$

$$B(r) \propto 1/r^2 \quad \text{for } \left(\frac{r}{a}\right)^2 \ll 1$$

$$B(r) \propto 1/r \quad \text{for } \left(\frac{r}{a}\right)^2 \gg 1$$

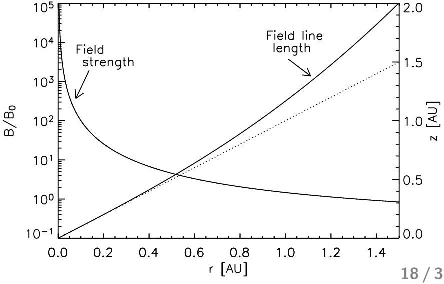


Table : Field line length and magnetic footpoint for two solar wind speeds

	$u = 400$ km/s		$u = 600$ km/s	
r (AU)	z (AU)	Footpoint	z (AU)	Footpoint
0.30	0.30	W18	0.29	W12
0.50	0.51	W30	0.50	W20
1.00	1.16	W61	1.07	W41

Propagation in the IMF

- In the interplanetary medium we consider an infinitely conducting plasma → Ohm's law: $\vec{E} = -\vec{u} \times \vec{B}/c$, where $\vec{u}$ is the solar wind speed

  $$\frac{d\vec{p}}{dt} = \frac{q}{c}(\vec{v} - \vec{u}) \times \vec{B}$$

  In a coordinate system where $\vec{u}||\vec{B}$ we can omit the solar wind speed from the equation of motion → GCs follow co-rotation

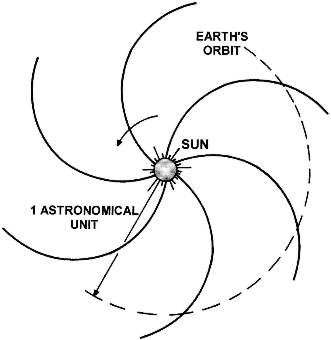
- Other drifts negligible in the inner heliosphere, but considerable for high energy protons

- Streaming along the field lines

- Focusing

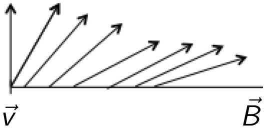
Focused motion along the Parker spiral

- 1st adiabatic invariant: $\frac{\sin^2\alpha}{B} = \text{const.}$
- Pitch angle α decreases when a particle moves in a weaker B $\longrightarrow$ The particle motion becomes more focused in the field direction (**focusing effect**)



- Particles released isotropically at the Sun appear to come in a narrow cone only $\sim 1^\circ$ wide at 1 AU

$\alpha_0 = 90^\circ$
$r_0 = 1R_\odot$



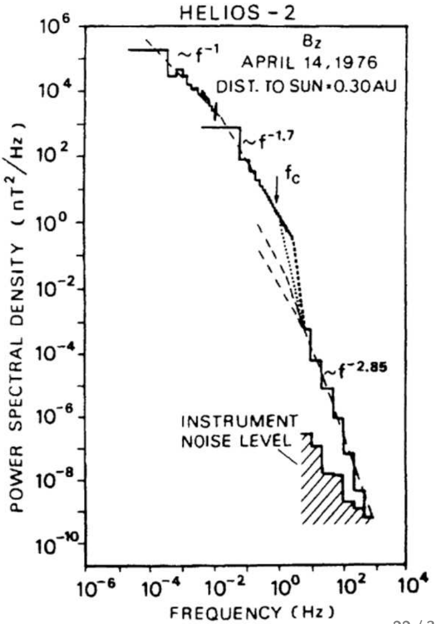
$\alpha' = 1^\circ$
$r' = 1$ AU

Fluctuations in the IMF

- The magnetic field fluctuations can be described by a power-density spectrum
  $$f(k_{||}) = Ck_{||}^{-q}$$
  $k_{||}$ is the wave number parallel to the field, q is the slope, and C a constant describing the level of turbulence.

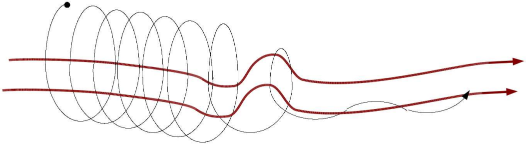
Inertial range (10^{-4}–1 Hz) mainly Alfven waves, $q \sim 1.5 - 1.9$

Dissipation range (above 1 Hz) whistlers, ion cycloton/acoustic waves, $q \sim 3$



Magnetic scattering

- Conservation of first adiabatic invariant requires smooth fields
- If $L_B \sim r_L$, the first adiabatic invariant is no longer conserved $\longrightarrow$ scattering

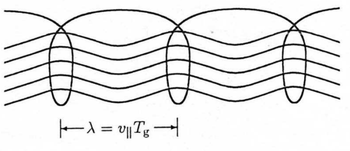


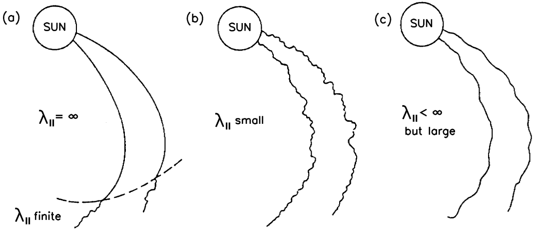
- Changes the pitch angle ($\longrightarrow$ pitch-angle diffusion)
- Changes the GC position ($\longrightarrow$ spatial diffusion $\perp \vec{B}$)
  - random walk (or meandering) of field lines
  - strongly turbulent fields may displace the GC as much as a Larmor radius

Under debate for some time!

Pitch-angle scattering

- Particles are scattered by magnetic irregularities which are in resonance with the particle gyration.
- As a cumulative result of many small random changes in pitch-angle, SEPs experience a macroscopic change in direction.
- An important special case which has been studied extensively is the quasilinear theory (QLT) of pitch-angle scattering and various modifications.



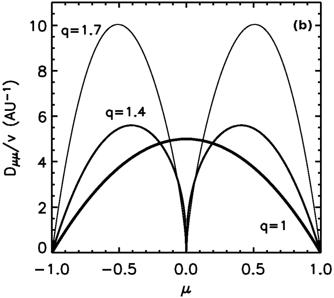


Pitch-angle diffusion coefficient

- Diffusion coefficient (Jokipii 1966)

- standard model of particle scattering
    - Small irregularities (QLT)
    - Transverse and axially symmetric fluctuations
- $P(k) \propto k^{-q}$

$$\boxed{D_{\mu\mu} = \frac{\nu(\mu)}{2}(1-\mu^2)} \; ; \; \boxed{\nu(\mu) = \nu_0|\mu|^{q-1}}$$



- Parallel mean free path (Hasselmann & Wibberenz 1968,1970)

$$\boxed{\lambda_{||} = \frac{3v}{8}\int_{-1}^{1}\frac{(1-\mu^2)^2}{D_{\mu\mu}}d\mu = \frac{3v}{4}\int_{-1}^{1}\frac{(1-\mu^2)}{\nu(\mu)}d\mu}$$

isotropic scattering $(\nu = \nu_0) \Rightarrow \lambda_{||} = \frac{v}{\nu_0}$

$\lambda_r = \lambda_{||}\cos^2\psi = \mathrm{const.}$ (Palmer 1982, Kallenrode et al. 1992, Ruffolo et al. 1998)

Transport models

- **Scatter-free transport** (the simplest approach)
  - SEPs stream away from the solar source without scattering
  - Focusing is very efficient near the Sun (field-aligned beam) → streaming along the field lines; IP transport takes $\Delta z / v$
  - But: (almost) never observed

- **Diffusion-convection approach** (Parker 1965) (the other extreme)
  - particle scattering efficient enough to keep the distribution almost isotropic
  - fluctuations move (diffusion wrt. the scattering centers (δB) → solar wind convection)
  - radial flow is diverging (expanding gas) → adiabatic cooling

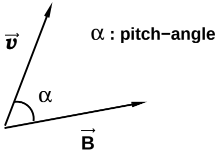
$$\frac{\partial f}{\partial t} + \mathbf{V} \cdot \nabla f - \frac{p}{3}(\nabla \cdot \mathbf{V}) \frac{\partial f}{\partial p} = \nabla \cdot (\mathbf{D} \cdot \nabla f)$$

$$f(\mathbf{r}, p, t) = \frac{1}{2\pi p^2} \frac{d^4 N}{d^3 r dp}$$

Focused transport

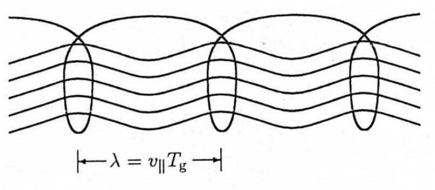
Focused transport equation (Roelof 1969)

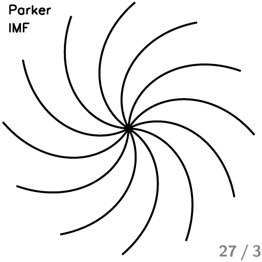
$$\frac{\partial f}{\partial t} + v\mu\frac{\partial f}{\partial z} + \frac{1-\mu^2}{2L}v\frac{\partial f}{\partial \mu} - \frac{\partial}{\partial \mu}\left(D_{\mu\mu}\frac{\partial f}{\partial \mu}\right) = q(z,\mu,t)$$



$$\mu = \cos\alpha = \frac{v_{||}}{v}$$

- Gyration around and streaming along the IMF
- Focusing and mirroring: $\frac{1-\mu^2}{B} = \mathrm{const.}$
- Diffusion in pitch-angle $\Longrightarrow$ spatial diffusion (scattering off magnetic irregularities)





Particle Transport Models

$$\frac{\partial f}{\partial t} + v\mu\frac{\partial f}{\partial z} + \frac{1-\mu^2}{2L}v\frac{\partial f}{\partial \mu} - \frac{\partial}{\partial \mu}\left(D_{\mu\mu}\frac{\partial f}{\partial \mu}\right) = q(z,\mu,t)$$

- Finite-difference numerical method:

  Ruffolo 1995, Lario et al. 1998, Hatzky & Kallenrode 1999, Dröge 2000

  ↑ Advantages: computationally fast

- Monte Carlo method:

  Kocharov et al. 1998, Zhang 2000, Li et al. 2003, Maia et al. 2007, Agueda et al. 2008

  ↑ Advantages: track of individual particles

Green's Functions of Particle Transport

Model Assumptions:

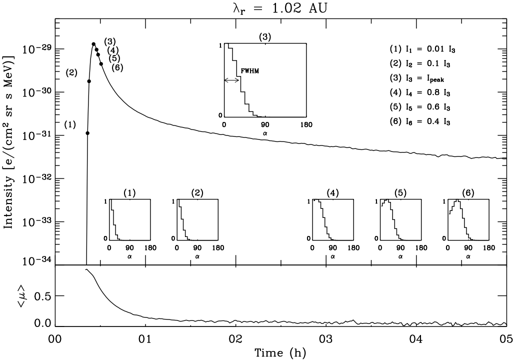
- Static source at $2R_\odot$
  Power spectra $\propto E^{-\gamma}$
- Archimedean IMF
- Scattering model

Model Parameters:

- Source spectral index (γ)
- Solar wind speed
- Diffusion coefficient ($D_{\mu\mu}$)
- Mean free path (λ_r)

The results of the simulations are

- differential intensities at the S/C location
- resulting from an instantaneous injection
- normalized to one particle injected per steradian



45-62 keV

Green's Functions of Particle Transport

Model Assumptions:

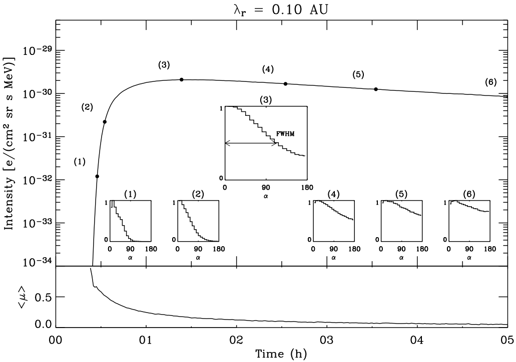
- Static source at $2R_\odot$
  Power spectra $\propto E^{-\gamma}$
- Archimedean IMF
- Scattering model

Model Parameters:

- Source spectral index (γ)
- Solar wind speed
- Diffusion coefficient ($D_{\mu\mu}$)
- Mean free path (λ_r)

The results of the simulations are

- differential intensities at the S/C location
- resulting from an instantaneous injection
- normalized to one particle injected per steradian

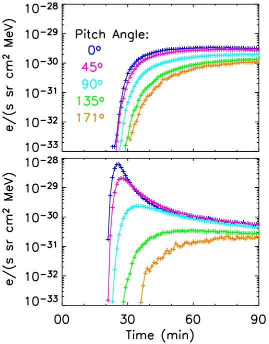


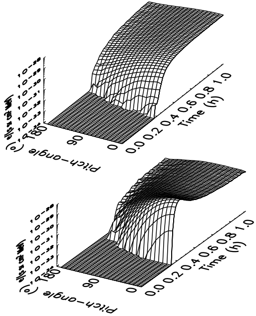
45-62 keV

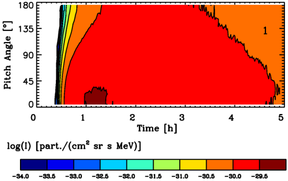
Green's Functions of Particle IP Transport

$\lambda_r = 0.10$ AU

$\lambda_r = 1.02$ AU



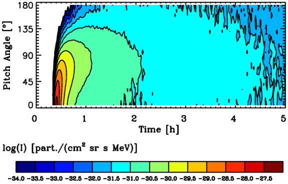




IP scenario:

Electron source at 0.01 AU,
$E^{-\gamma}$ with γ=3.0, 24–400 keV
V_{sw}=400 km/s
λ_r=0.10 AU; Isotropic

Generated by SEPServer



IP scenario:

Electron source at 0.01 AU,
$E^{-\gamma}$ with γ=3.0, 24–400 keV
V_{sw}=400 km/s
λ_r=1.02 AU; Isotropic

Generated by SEPServer

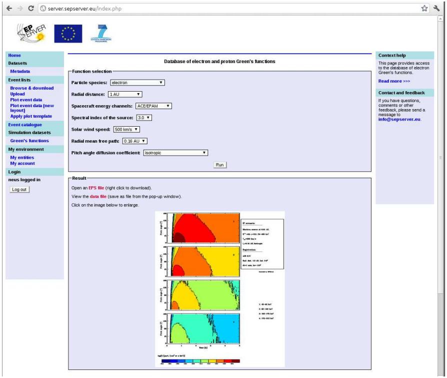
SEPServer Database of Simulation Results

Database of Green's functions available through the SEPServer website!

Selection:

1. Particle specie (electron, proton, relativistic particle)
2. Transport scenario (γ, u, λ_r, $D_{\mu\mu}$)
3. Registration bins (or S/C)

`http://server.sepserver.eu`



Database of electron and proton Green's functions

Function selection

Particle species: electron

Radial distance: 1 AU

Spacecraft energy channels: ACE/EPAM

Spectral index of the source: 3.0

Solar wind speed: 500 km/s

Radial mean free path: 0.16 AU

Pitch angle diffusion coefficient: isotropic

Result

Open an EPS file (right click to download).

View the data file (save as file from the pop-up window).

Click on the image below to enlarge.

Context help

This page provides access to the database of electron Green's functions.

Read more >>>

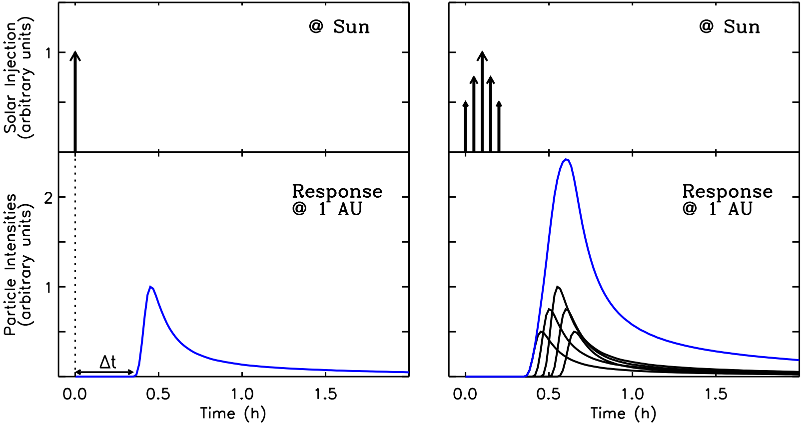
Contact and feedback

If you have questions, comments or other feedback, please send a message to info@sepserver.eu.

Download:

- Results (text data file or EPS plot)
- Documentation

The Power of Convolution



$$\boxed{I(t) = \int_0^\infty q(t')g(t-t')dt'}$$

Injection Function

Green's function
λ_r; (t,α,E)

- **Forward Modeling:** Prediction of the measurements with a given set of model parameters. Inductive.
    - ↓ Trial and error. Difficult to scan all the parameter's space

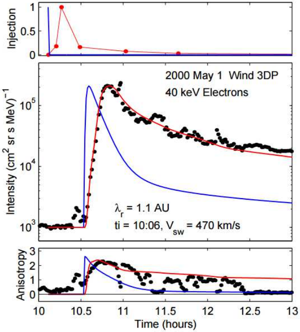
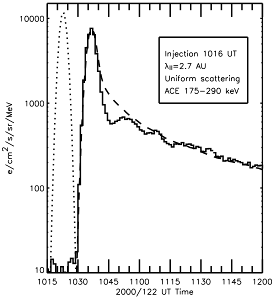
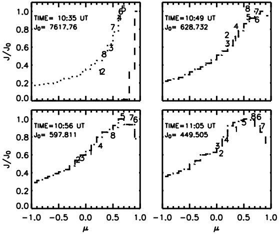


Figure 1: Fits to the time-intensity and anisotropy profiles of 40 keV electrons observed on Wind after the 2000 May 1 flare, assuming a constant $\lambda_r = 1.1$ AU.



Kartavykh et al. (2008)



Agueda et al. (2008)

Methods for Data Fitting

- **Inverse Modeling:** Use of the measurements to infer the actual values of the model parameters. Deductive.
    - ↑ Systematic exploration of the parameters space. Reproducible.
    - ↑ No a priori assumption about the injection profile.

The injection function can be determined by solving the least squares problem

$$||\vec{J} - \mathbf{g} \cdot \vec{q}|| \sim 0$$

Observations

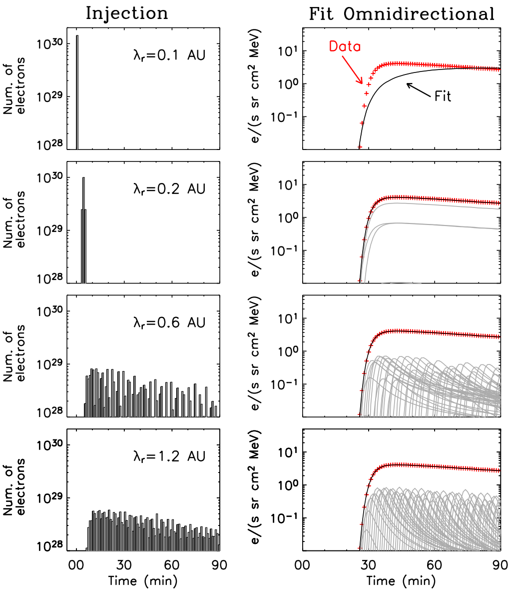
Modeled intensities

subject to the constraint that $q_j \geq 0 \; \forall j$ (NNLS; Lawson & Hanson 1974)

The problem is ill-posed if $\vec{J}$ are omni-directional intensities

The problem is well-constrained if $\vec{J}$ are directional intensities

When is the problem ill-posed?

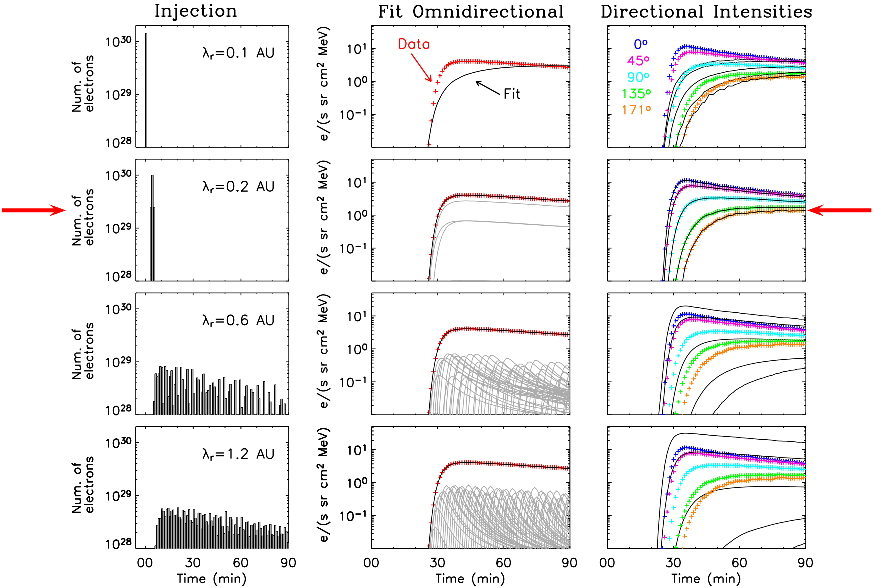


Too much freedom!

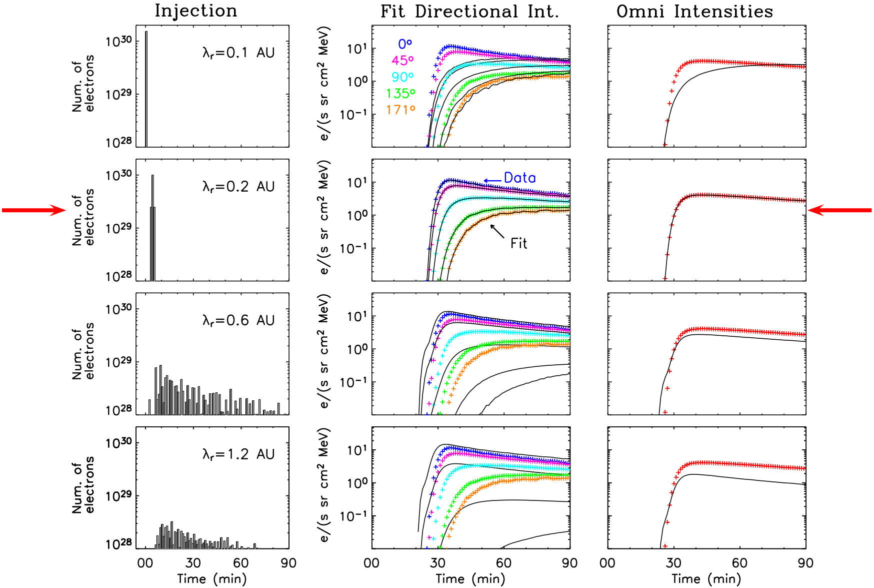
More constraints:

a) 1st order anisotropy
b) Directional Intensities

When is the problem ill-posed?



Inversion of Directional Intensities



Take away message

- The average interplanetary magnetic field can be described by an Archimedean spiral.

- Superposed on this average spiral field are small-scale random fluctuations. Further, the individual field lines random-walk (or meander) relative to the average direction.

- In the co-rotating frame, the motion of the GC of SEPs travels along the mean spiral direction. The next higher approximation takes into account that particles are scattered by small-scale fluctuations in the field that are embedded in the solar wind. Higher order approximations consider drifts due to curvature and density gradients, and the random walk of the field lines.